Normal mode damping continued, Structural Vibes

$$M\ddot{\vec{x}} + C\dot{\vec{x}} + K\vec{x} = \vec{F}(t)$$

-try to decouple the equations into separate 1-DoF systems

1.) $\vec{x} = M^{-1/2}\vec{q}$

$$\ddot{\vec{x}} + M^{-1/2} C M^{-1/2}\dot{\vec{q}} + \underbrace{M^{-1/2} K M^{-1/2}}_{\tilde{K}}\vec{q} = M^{-1/2}\vec{F}(t)$$

$\hookrightarrow P =$ e-vectors of $\tilde{K}$ (normalized)

$\vec{q} = P\vec{r}$

$\hookrightarrow$ normal mode shapes $\longrightarrow$ substitute in and multiply by P^{-1}

$\hookrightarrow = P^T$ (symmetry)

$$\ddot{\vec{r}} + \underbrace{P^T M^{-1/2} C M^{-1/2} P}_{\substack{\tilde{C} \\ \text{(mess)}}}\dot{\vec{r}} + \Lambda\vec{r} = P^T M^{-1/2}\vec{F}$$

$\hookrightarrow \begin{bmatrix} \lambda_1 & 0 & 0 & 0 & \cdots \\ 0 & \lambda_2 & 0 & 0 & \cdots \\ 0 & 0 & \lambda_3 & 0 & 0 \end{bmatrix}$

$\tilde{C}$ is a mess

$$\ddot{r}_? + [\tilde{C}\dot{\vec{r}}]_? + \omega_?^2 r_? = [P^T M^{-1/2}\vec{F}]_? \quad * \text{ (decoupled but for damping terms)}$$

$\hookrightarrow ? = i/j/k/n$, etc

$\rightarrow$ can't decouple the equations

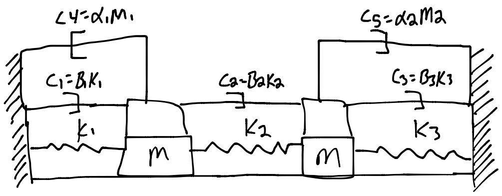
$\rightarrow$ wish the problem away

Method 1: Assume (throw away info) $C = \alpha M + \beta K$

gives: $$\ddot{\vec{r}} + (\alpha I + \beta\Lambda)\dot{\vec{r}} + \Lambda\vec{r} = \tilde{\tilde{F}}$$

$$\ddot{\vec{r}}_7 + (\alpha + \beta \omega_7^2)\,\dot{\vec{r}}_7 + \omega_7^2\,\vec{r}_7 = [\tilde{\tilde{F}}]_7$$

Example:



add dashpots next to every spring with $(c_i = \beta_i k_i)$

add dashpots to ground for every mass with $c_j = \alpha m_j$

α affects low frequency modes, and has to do with the overall motion

β affects the high frequency modes, and has to do with relative motion

- look at a system and try to pick α and β

Method 2: $P^{-1} M^{-1/2} C M^{1/2} P = \tilde{C}$

$\tilde{C} = [\ \ ]$

- cross off off-diagonal terms

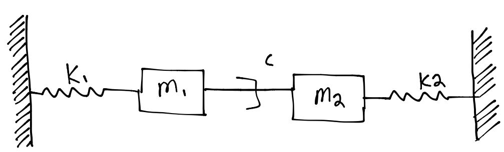
a.) Given model, set $\tilde{C}_{ij} = 0$ if $i \neq j$

b.) Experiment in system with no model

* excite one mode; * pick damping for that mode, call it $\tilde{C}_i$

* look for its rate of decay

Example: bad case



C is not α or β form

- two modes are coupled

approximate as coupled